Financial Crisis and Global Market Couplings

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***Abstract*—The global financial crisis occurred in 2007 and its severe damaging consequences on other global financial markets, show the great importance of understanding the impact and contagion between different financial markets. A variety of methods have been proposed and implemented on market contagion. However, most of the existing literature simply test the existence of market contagion in financial crisis, and there is limited work go deep to investigate the complex market couplings which are the essence of market contagion. This is indeed very difficult as it involves the selection of discriminative indicators, the different types of couplings (intra-market coupling, inter-market coupling), the hidden characteristic of couplings, and the evaluation of market couplings in understanding crisis. To address these issues, this paper proposes a CHMM-LR framework to investigate the relations between financial crisis and three pairwise market couplings from three typical global financial markets: Equity market, Commodity market and Interest market. We adopt Coupled Hidden Markov Model (CHMM) to capture the complex hidden pairwise market couplings, and the financial crisis forecasting abilities based on different pairwise market couplings are imported to measure the relations by Logistic Regression (LR). Experiments of real financial data during the period 1990 to 2010 show the advantages of market couplings in understanding crisis. In addition, the experimental results provide crucial interpretation for the 2008 global financial crisis periods identification.**

I. Introduction

The subprime mortgage crisis which occurred in the US in 2007 has caused a chain of destructive effects on global financial markets. During the crisis, the issues of risk management and asset allocation in different markets are very important to investors and researchers. So the study of global markets contagion behaviors is of great importance as it can provide information about how crisis spread. Here market contagion refers to the correlations/transmissions from one market to another.

A large body of literature have investigated the contagion analysis. Authors in [1] analyze the contagion effects from five emerging equity markets during global financial crisis, and find mixed evidence for contagion. Contagion effects are verified by both equity and bond markets of emerging economies around the world during the 2007 subprime crisis in [2]. However, most of the existing literature focus on testing market contagion, and most of them provide evidence on the correlations/transmissions of financial markets during financial crisis [3], [4]. To the best of our knowledge, limited research pay attention to the unknown underlying market couplings which are the "fundamental" reasons for the market contagion [5]. Market Couplings here refer to the interaction between behaviors across different markets (c.f. Section III). The crisis effect passed from one market to another through the couplings and reflected by the market indicators (e.g. market index). Once we captured the complicated couplings, we can obtain more information about the crisis. In addition, the existing literature mostly concentrates on equity markets and studies based on other global financial markets (e.g. commodity market and interest market) are rare [6].

The above issues suggest the need to investigate the relations between market couplings and crisis across different types of global financial markets. However, it is not a trivial task and the difficulties may lie in following four aspects: (1) the selection of global markets and discriminative market indicators; (2) the market couplings are very complicated to capture, which includes intra-market coupling (couplings in one market) and inter-market coupling (couplings between different types of markets); (3) the couplings are hidden behind the market indicators; (4) the measurement of the effects of market couplings in understanding crisis, namely how the couplings contagions reflect the crisis.

To address these issues, in this paper, we investigate the relations between financial crisis and couplings from three typical global financial markets [7], namely Equity market, Commodity market and Interest market, from 1990 to 2010. We test various pairwise market couplings, that is: (1) couplings from Equity market and Commodity market ($\mathbb{C}(E,C)$); (2) couplings from Equity market and Interest market ($\mathbb{C}(E,I)$); (3) couplings from Commodity market and Interest market ($\mathbb{C}(C,I)$). Furthermore, to better evaluate the capacity of market couplings in understanding crisis, here we use the different pairwise market couplings to forecast global crisis, namely using the crisis forecasting abilities as the measurement. Based on this, we build a CHMM-LR framework to investigate the relations. As a machine learning-based method, Coupled Hidden Markov Model (CHMM) is applied for its power at modeling complicated coupling processes [8], and Logistic Regression (LR) is a typical time series forecasting model. The working process is as follows: with the pairwise markets, we first learn the corresponding market couplings through CHMM model. Subsequently, the learned couplings are fed into LR model to forecast crisis. Then, we investigate the relations between the crisis and pairwise market couplings, by analyzing the forecasting behaviors obtained in the forecasting stage.

The rest of the paper is organized as follows. In Section II, we briefly introduce CHMM and LR models which are related to this paper. The corresponding problem is defined in Section III. Section IV provides the modeling framework and the corresponding three specific steps. The empirical results and their interpretation are displayed and discussed in Section V, while Section VI reports the summary and concluding remarks.

II. Technical Foundation

In this section, we briefly introduce the two models related to this paper, where CHMM is built to capture the hidden couplings across various global markets, and LR is imported to forecast crisis.

A. Coupled Hidden Markov Model

CHMM [9] was proposed to model multiple processes with coupling relationships. CHMM consists of more than one chain of HMMs, and each HMM represents one process. In CHMM, the state of any chain of HMM at time t depends not only on the states of its own chain, but also the states of other chains of HMMs at time $t-1$. These are namely the interactions between the processes. Fig. 1 is a standard CHMM with two chains. The hidden variables Z are assumed to interact locally with their neighbors and each of them has its own observation O.

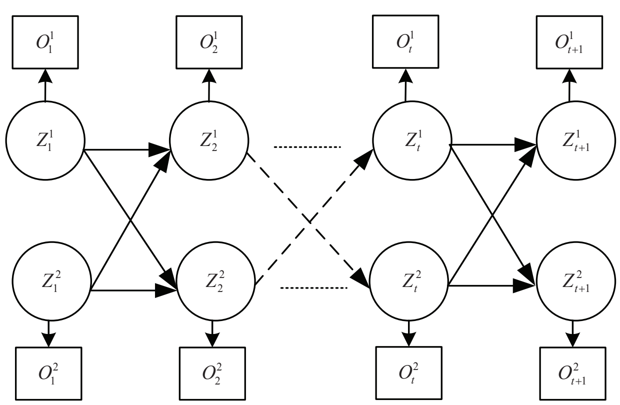


Fig. 1: A CHMM with Two Chains

Suppose there are N coupled HMMs, H is the number of states of the Markov chains, $\{Z_1, Z_2, \ldots, Z_H\}$ is a set of hidden states, where z_t is the hidden state at time t. V is the number of observation symbols, $\{X_1, X_2, \ldots, X_V\}$ is a set of observation symbols, $O = \{O_1, O_2, \ldots, O_T\}$ is an observation sequence, o_t is the observation at time t. And the corresponding elements of a CHMM can be defined as follows:

The number of Markov chains N, they are coupled with each other.

A set of hidden states $\{Z_1, Z_2, \ldots, Z_H\}$, where H is the number of hidden states.

A set of observation symbols $\{X_1, X_2, \ldots, X_V\}$, where V is the number of observation symbols.

Prior probability of initial state $\pi = \{\pi_h^{(n)}\}, 1 \leq n \leq N, 1 \leq h \leq H^{(n)}$
$\pi_h^{(n)} = p(z_1^{(n)} = Z_h)$, s.t. $\sum_{h=1}^{H^{(n)}} \pi_h^{(n)} = 1$

State transition probability matrix $A = \{a_{h'h}^{(n',n)}\}, 1 \leq n', n \leq N, 1 \leq h', h \leq H^{(n)}$
$a_{h'h}^{(n',n)} = p(z_{t+1}^n = Z_h | z_t^{n'} = Z_{h'})$, s.t. $\sum_{h=1}^{H^{(n)}} a_{h'h}^{(n',n)} = 1$
Observation probability matrix $B = \{b_h^{(n)}(v)\}, 1 \leq n \leq N, 1 \leq h \leq H^{(n)}, 1 \leq v \leq V$
$b_h^{(n)}(v) = p(o_t^{(n)} = X_v | z_t = Z_h)$, s.t. $\sum_{v=1}^{V} b_h^{(n)}(v) = 1$

Coupling coefficient $R = r_{n'n}, 1 \leq n', n \leq N$
s.t. $\sum_{n'=1}^{N} r_{n'n} = 1$

For convenience, similar to [10], we refer to the complete set of parameters of a CHMM as $\lambda(A, B, R, \pi)$.

B. Logistical Regression

The Logistic model [11] is a direct probability model which measures the relationship between the categorical dependent variable and one or more independent variables, that are usually (but not necessarily) continuous. Based on the relationship, the model can predict the outcome of the categorically dependent variable at future time periods.

In this paper, we distinguish between two categories: crisis and non-crisis. Suppose $Y_t = 1$ represents a crisis at time t, and $Y_t = 0$ indicates non-crisis. P_t is the probability of having a crisis at time t,

$$P_t = P(Y_t = 1 | X = x) = E(Y_t | X = x) = \frac{1}{1 + e^{-b_0 + b_1 x_1 + \cdots + b_n x_n + \varepsilon}} \quad (1)$$

here $x_i (1 \leq i \leq n)$ is explanatory variable, ε is the error term. Then the likelihood function can be written as follows:

$$L(\theta) = \prod_{t=1}^{T} {P_t}^{Y_t} {(1 - P_t)}^{1 - Y_t} \quad (2)$$

where T is the number of periods. Then the parameters can be obtained through Maximum-Likelihood Estimation (MLE).

III. Problem Formalization

This paper aims to investigate the relations between different pairwise market couplings and financial crisis, we here first introduce some concepts related to market coupling and then formalize the problem.

Definition 1. ***Intra-market Coupling***. *This is the interaction between the behaviors from the same market. Formally, the* *representation of intra-market coupling w.r.t market i is given by:*

$$\theta_i = \{m_i \otimes m_i\}_{t=1}^{T} \tag{3}$$

where m_i denotes the observations from market i, $\otimes$ represents the coupled interactions among market i's observations from time 1 to T. In this paper, there are three global financial markets, so $i \in \{E, C, I\}$.

Definition 2. ***Inter-market Coupling***. *This is the interaction between the behaviors from pairwise markets. Formally, the representation of inter-market coupling w.r.t market i and j is given by:*

$$\eta_i = \{m_i \circledast m_j\}_{t=1}^{T} \tag{4}$$

where $\circledast$ represents the coupled interactions between market i's observations and market j's observations from time 1 to T, $i, j \in \{E, C, I\}$.

Definition 3. ***Market Coupling***. *The representation of market coupling w.r.t market i and j is given by:*

$$\mathbb{C}(i,j) = \{\theta_i, \eta_{ij}\} \tag{5}$$

where θ_i denotes the intra-market coupling in market i, and η_{ij} represents the inter-market coupling between markets i and j.

Then, the problem can be formalized as following: $\mathbb{C}(i,j)$ is used to capture the complex pairwise couplings between markets i and j. Let $R(\cdot)$ be the objective function to evaluate the relations between different pairwise market couplings and crisis, namely evaluating the financial crisis forecasting abilities of various market couplings. So at different time period t, the motivation is:

$$argmax_{(i,j)} R(crisis, \mathbb{C}(i,j)) \tag{6}$$

The key task then is to build a proper model to determine the specific pairwise coupling $\mathbb{C}_(i,j)$ and the corresponding objective function $R(\cdot)$. Below, CHMM is explored to capture the complex coupling relationships and nonlinear dynamics of pairwise markets, with LR to measure the forecasting abilities based on pairwise couplings obtained from CHMM.

IV. Modeling Framework

Based on the problem definition in Section III, we propose a framework based on CHMM and LR, which is depicted in Fig. 2. It consists of the following three major parts: 1) Index selection, which selects one proper indicator for each global financial market that better fit the CHMM analysis; 2) Coupling process, namely exploring CHMM to capture the complex hidden couplings $\mathbb{C}(i,j)$ between pairwise global markets i and j. For example, coupling between Equity market and Interest market $\mathbb{C}(E,I)$; 3) Forecasting process, which imports LR to evaluate the global financial crisis forecasting abilities based on the obtained pairwise market couplings.

A. Indicator Selection

As illustrated in TABLE I, there are more than one typical indicator for each global financial market. However, we use one Markov chain to represent a market at coupling process. To better fit the CHMM model, here we select one indicator for each market which has the highest correlations with another market. This is because our focus is on the couplings between pairwise markets, hence higher correlation with another market encloses a stronger discriminative power.

Definition 4. ***Pairwise Market Indicator Correlation (PMIC)***. *This is the correlation of one indicator in a market (MI_{ik}) with indicators in another market ($\{MI_{jl}\}$), where $(i \neq j) \wedge (i, j \in \{E, C, I\}) \wedge (k, l \in \{1, 2\})$.*

$$PMIC(MI_{ik}, \{MI_{jl}\}) = \sum_{l} \mid \rho_{DCCA}(MI_{ik}, MI_{jl}) \mid \tag{7}$$

where $\rho_{DCCA}(\cdot)$ is the cross-correlation coefficient of the two market indicators.

Here we use the Detrended cross-correlation analysis (DCCA) [12] to quantify the cross-correlations between two non-stationary time series (market sequences). ρ_{DCCA} is the DCCA cross-correlation coefficient proposed in [13], which can calculate the level of cross-correlations between two non-stationary time series. It is a dimensionless coefficient that ranges between [-1,1]. If two time series are completely cross-correlated (anti cross-correlated) then $\rho_{DCCA} = 1(-1)$, and if there are no cross-correlation between two time series then $\rho_{DCCA} = 0$. More details refer to [13], [14].

Then between two markets i and j, we select the indicator which has biggest value of PMIC to represent market i:

$$argmax_k PMIC(MI_{ik}, \{MI_{jl}\}) \tag{8}$$

where $k \in \{1, 2\}$ since there are two typical indicators for each market.

B. Coupling Process

In this paper, we investigate three global financial markets: Equity market, Commodity market and Interest market, and we select one indicator for each market. Correspondingly, there are three Markov chains, namely HMM-E enclosing the Equity market sequence $\Phi(MI_S)$, HMM-C capturing the Commodity market sequence $\Phi(MI_C)$, and HMM-I for the Interest market sequence $\Phi(MI_I)$. Then, for each pair markets i and j, we build one CHMM to incorporate corresponding two market sequences, which maps the observations into hidden couplings. Below is the specific mapping process:

$$Pairwise\, Market\, Couplings \rightarrow CHMM\, modeling$$

$$\Phi(MI^i)|observation \rightarrow B(P(o_t^i = X_v | z_t^i = Z_h)) \tag{9}$$

$$\Phi(MI^i)|intra-transition(\theta_i) \rightarrow A|intra(P(z_{t+1}^i = Z_{h'} | z_t^i = Z_h)) \tag{10}$$

$$\Phi(MI^i), \Phi(MI^j)|inter-transition(\eta_{ij}) \rightarrow A|inter(P(z_{t+1}^i = Z_{h'} | z_t^j = Z_h)) \tag{11}$$

$$\mathbb{C}(i,j) \rightarrow \{z^i, z^j\} \tag{12}$$

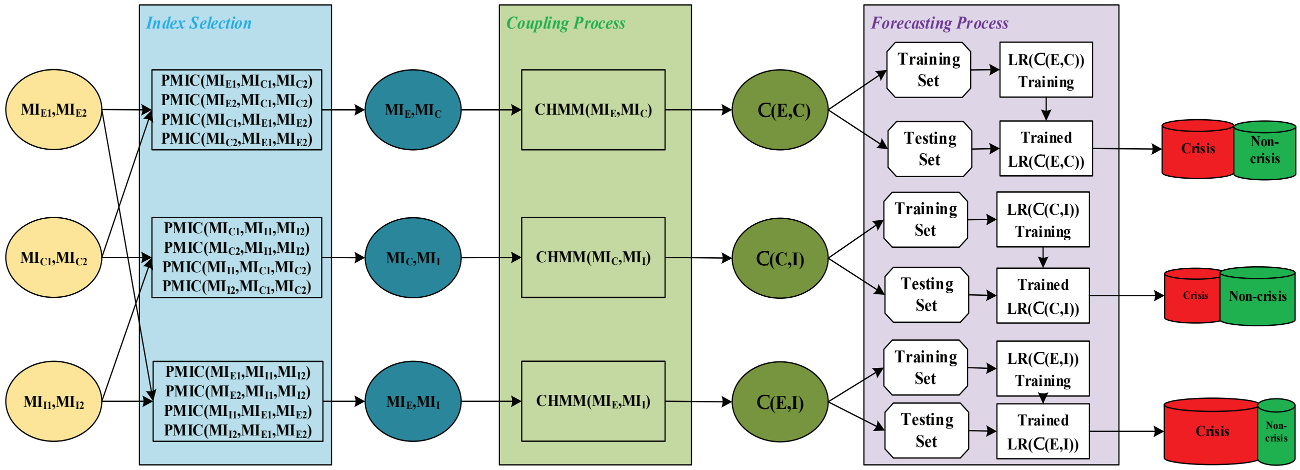


Fig. 2: The Proposed Framework

TABLE I: Trading Indicators

Market	Indicator	Description
Equity	S&P 500	An index based on 500 leading companies publicly traded in the US equity market.
	DJIA	An index based on 30 large publicly owned companies in the US equity market.
Commodity	Gold price	An indicator that often used as refuge for asset safety during financial crisis periods.
	WTI Crude Oil Price	An indicator that becomes a major commodity market.
Interest	TED Spread	The difference between the 3-month interest rates on interbank loans and on 3-month Treasury bill rate.
	BAA Spread	The difference between the Baa Corporate bond rate and 10 year Treasury bill rate.

where $(i \neq j) \wedge i,j \in \{E,C,I\}$, $\{Z_1, Z_2, \ldots, Z_H\}$ is a set of hidden states, where z_t is the hidden state at time t. $\{X_1, X_2, \ldots, X_V\}$ is a set of observation symbols, o_t is the observation at time t.

Based on the above discussion, we can easily find that the intra-market coupling in Equation (3) is illustrated by $P(z^i_{t+1} = Z_{h'}|z^i_t = Z_h)$, inter-market coupling is encoded with $P(z^i_{t+1} = Z_{h'}|z^j_t = Z_h)$. Therefore, $\{z^i, z^j\}$ serves as the hidden coupling between market i and market j ($\mathbb{C}(i,j)$). Below we briefly discuss the computation of state transition probability $P(z^i_{t+1}|z^i_t, z^j_t)$.

Suppose there are N chains (in this paper $N = 2$), each Markov chain owns H hidden states, then the state transition probability is

$$P(z^n_t \mid z^1_{t-1}, z^2_{t-1}, \ldots, z^N_{t-1}) \tag{13}$$

where the $z^{(n)}_t$ is the hidden state of chain n at time t, then the number of parameters to calculate the state transition probability is H^N. To learn the parameters, many researchers proposed several variations of CHMM. For instance, in [15], the state transition probability is defined as the product of all marginal conditional probabilities:

$$P(z^n_t \mid z^1_{t-1}, z^2_{t-1}, \ldots, z^N_{t-1}) = \prod_{n'=1}^{N} P(z^n_t \mid z^{n'}_{t-1}) \tag{14}$$

Then the transition probability parameter space is reduced [16]. However, as illustrated in [17], Equation (14) does not hold since the right hand side does not equal to one. In this paper, we follow the method proposed by Zhong in [10], which models the joint transition probability as:

$$P(z^n_t \mid z^1_{t-1}, z^2_{t-1}, \ldots, z^N_{t-1}) = \sum_{n'=1}^{N} (r_{n'n} P(z^n_t \mid z^{n'}_{t-1})) \tag{15}$$

where the joint transition probability is modeled as a linear combination of various marginal conditional probabilities. Here $r_{n'n}$ is the coupling coefficient which evaluates the coupling weight from model n' to model n, the bigger the $r_{n'n}$, the more $z^{n'}_{t-1}$ affects z^n_t.

Based on above, in this paper the state transition probability $P(z^i_{t+1}|z^i_t, z^j_t)$ is calculated as:

$$P(z^i_{t+1}|z^i_t, z^j_t) = r_{ii} P(z^i_{t+1}|z^i_t) + rij P(z^i_{t+1}|z^j_t) \tag{16}$$

where $i,j \in \{E,C,I\}$. Then the corresponding algorithm in [10] is applied to learn the parameters.

C. Forecasting Process

The above section explores the couplings $\mathbb{C}(i,j)$ between pairwise global financial markets i and j through CHMM. Below we discuss the evaluation of relations of various market couplings and financial crisis, namely calculating the financial crisis forecasting abilities based on the different types of couplings. Here we import the classical LR model to do the forecasting, the general framework of proposed forecasting process is illustrated in Fig. 3.

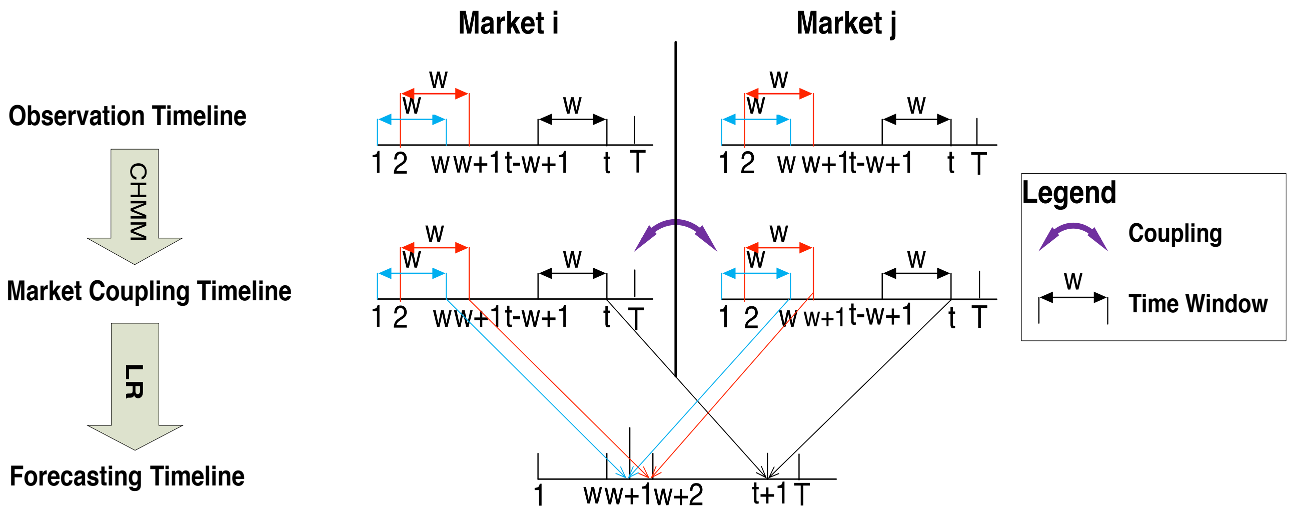


Fig. 3: Forecasting Process

For each observation interval $[t-w+1, t]$ (w is the time window, $t \in [w, T]$), the first step is to train the CHMM using the w observations ($O^i_{t-w+1:t}, O^j_{t-w+1:t}$) from the pairwise financial markets i and j to obtain corresponding market couplings $\{z^i_\tau, z^j_\tau\}^t_{\tau=t-w+1}$) (illustrated in coupling process). Then based on the couplings, a trained Logistic Regression Model (trained from training set) gives the probability of crisis ($P_{t+1}(crisis = 1|\{z^i_\tau, z^j_\tau\}^t_{\tau=t-w+1})$) based on the pairwise market couplings.

Once the probability of crisis at time $t+1$ is obtained, we further determine whether the specific time $t+1$ is crisis or not through comparing the probability with corresponding threshold value. The corresponding algorithm is described in Algorithm 1. The input is the coupling from pairwise markets i and j $\{z^i_\tau, z^j_\tau\}^t_{\tau=t-w+1}$) where $i, j \in \{E, C, I\}$, and the time interval is w. Steps 1 is to train the Logistic Regression model Ω^{Tr} based on training set. Steps 2 to 10 form a loop process to compute the probability of crisis based on the trained model and couplings in testing set. The output of the algorithm includes two sets: crisis set $CS^{(i,j)}$ and non-crisis set $NS^{(i,j)}$.

Based on above discussion, for market i and j, we can obtain a predicted set $\hat{P}^{(i,j)} = \{CS^{(i,j)}, NS^{(i,j)}\}$, then the objective function $R(crisis, \mathbb{C}(i,j))$ in Equation (6) can be implemented by:

$$\Gamma^{(i,j)}(\hat{P}^{(i,j)}, \mathbb{P}) \tag{17}$$

and the motivation becomes:

$$argmax_{(i,j)} \Gamma^{(i,j)}(\hat{P}^{(i,j)}, \mathbb{P}) \tag{18}$$

where $\mathbb{P} = \{CS^{true}, NS^{true}\}$ is the true set of crisis and non-crisis, $\Gamma^{(i,j)}$ is a fit function to evaluate the fitting degree of predicted set and true set. The specific form of $\Gamma^{(i,j)}$ will be introduced in Section V.

Algorithm 1: Financial Crisis Forecasting via Market Couplings

Input: A training set Tr ; A testing set $Te = \{\{z^i_1, z^j_1\}, \{z^i_2, z^j_2\}, \ldots, \{z^i_t, z^j_t\}, \ldots \{z^i_T, z^j_T\}\}$
Output: A predicted financial crisis set CS; A predicted non-financial crisis set NS

1 Train the Logistic Regression model Ω on the training set Tr, obtained trained model Ω^{Tr};
2 **forall the** $\{z^i_\tau, z^j_\tau\}^t_{\tau=t-w+1} and\, t \in [w, T]$ *in the Testing set* **do**
3   Compute the probability of crisis given the trained model Ω^{Tr} and couplings $\{z^i, z^j\}^t_{\tau=t-w+1}$):
4     $P_{t+1}(crisis = 1|\{z^i_\tau, z^j_\tau\}$;
5   **if** $P_{t+1}(crisis = 1|\{z^i_\tau, z^j_\tau\}) > 0.5$ **then**
6   time $t+1 \to CS^{(i,j)}$;
7   **else**
8   time $t+1 \to NS^{(i,j)}$;
9   **end**
10 **end**

V. Empirical Analysis

In this section, we evaluate the relations of global financial crisis and various pairwise market couplings on real financial markets data and compare the results.

A. Experimental Setup

1) Data and Preliminary Analysis: This paper investigates the financial crisis with three pairwise market couplings. Thus, the data come from three global financial markets: Equity market, Commodity market and Interest market are extracted for the experiments. All data comprises weekly closing prices from January 1990 to December 2010, sourced from the Economic Research (http://research.stlouisfed.org/). As discussed in Section IV, for each pairwise market coupling, we only choose one indicator for a market, according to its correlations with another market. Based on Equation (7), the selected indicators for the three markets in different pairwise couplings are depicted in TABLE II .

TABLE II: Selected Indicators

Pairwise Coupling	Market Indicator
$\mathbb{C}(E, C)$	E: DJIA /C:WTI Oil Price
$\mathbb{C}(C, I)$	C: Gold Price/C:TED Spread
$\mathbb{C}(E, I)$	E: DJIA /I:BAA Spread

To better fit the model, the index prices are decoded into return as the symbols which can be calculated by

$$RI_t^i = \frac{PI_t^i - PI_{t-1}^i}{PI_{t-1}^i} * 100\%, \tag{19}$$

where RI_t^i and PI_t^i are, respectively, the return and closing price at time t in market i.

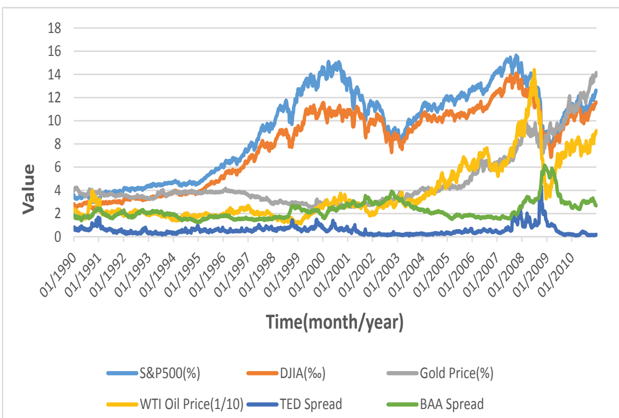


Fig. 4: Indicator behavior During the Period 1990-2010

Fig. 4 and Fig. 5 provide the graphic representation of index evolution and returns behavior of the selected indicators during the period from January 1990 to December 2010. From the figures we can easily find that the correlations of various indicators are not stable; there exists some hidden coupling behind the observations. In addition, both indexes and returns show a large fluctuation zone around 2008, which is the US sub-prime crisis trigged by Lehman Brother failure. This provides the evidence that the hidden couplings behave differently in and out crisis periods.

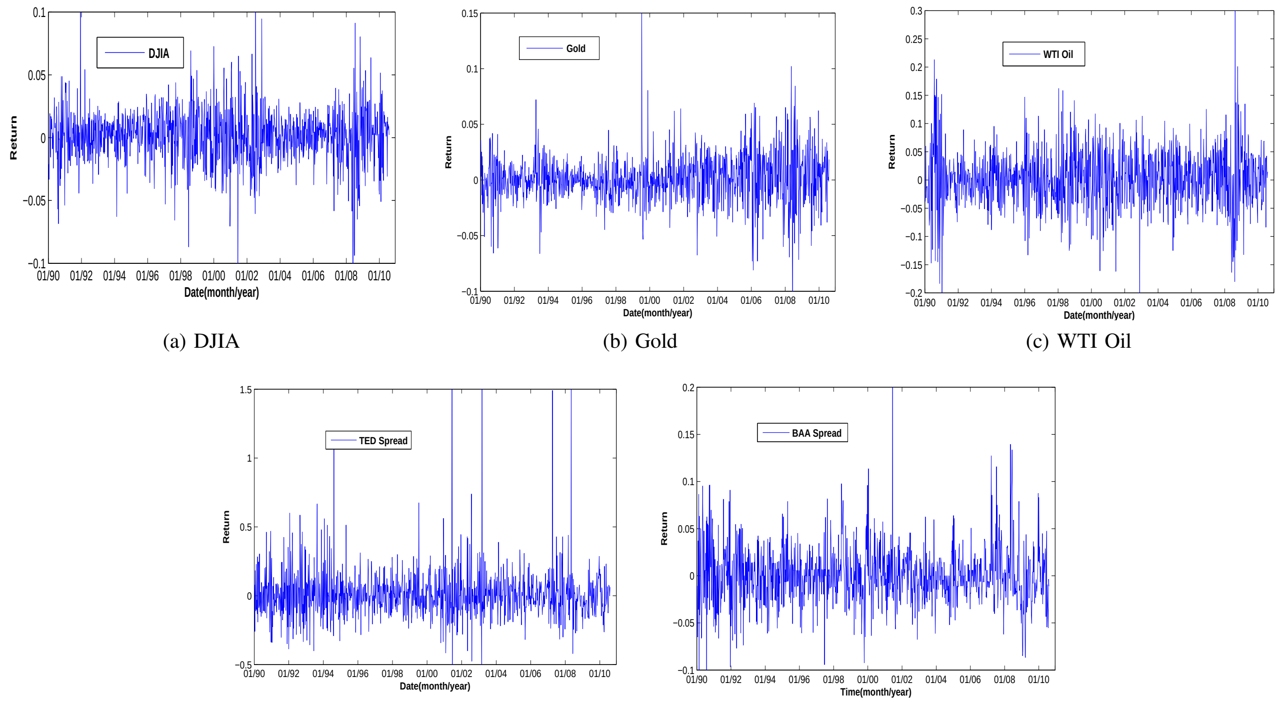
In the experiments we divide the data into two parts: training set from Jan 1990 to Dec 2006, testing set from Jan 2007 to Dec 2010. Domain knowledge from the National Bureau of Economic Research (NBER) Business Cycle Dating Committee (http://www.nber.org/cycles.html) is involved in this data splitting. According to the domain knowledge, there are two crisis periods in training data set: July 1990 to March 1991 (led by the Gulf war), and March 2001 to Nov 2001 (triggered by the dot-com bubble), one crisis period in testing data set: Dec 2007 to June 2009 (caused by the subprime crisis). As indicators in different markets may appear on different trading days, we delete those days on which some market data is missing and only choose the days with trading data from all global financial markets.

2) Parameter Specification: The parameter specification involves three steps. The first and second deal with the CHMM elements and CHMM initial values in coupling process, separately, and the third defines the element in LR forecasting process. The following paragraphs detail these three steps.

- *Specification of the CHMM Elements.* As mentioned in Section IV, there are three different pairwise market couplings, so the total number of CHMM is three. For each CHMM, there are two HMM sequences represent the two pairwise financial markets, so the number of Markov chains for each CHMM is two. In this paper, the number of states H is set equal to 10 based on the tests in experiments.
- *CHMM Initial Parameter Settings.* CHMM parameter estimation is done using the EM algorithm. Good starting values for parameters in the algorithm can help in speeding up the algorithm and ensuring promising results. Several possible kinds of initialization have been proposed. Using random starting values for the parameters and starting the algorithm from several different starting points and then selecting a better one is often used by researchers [18]. Here the initial parameter value of π and A follow the random selected method.
  The parameter which needs an initial value here is B, namely the Observation probability. To this end, we need to find the corresponding number of the mixture component. This is because, in an infinite Gaussian mixture model, how to find the right number of the mixture components is an important and difficult issue, and the right number will help obtain promising results [19]. As shown in Fig. 6 (here we use the selected DJIA indicator as an example), the distribution of the index is a mixture Gaussian, and we need to find the number of the mixture component. Based on former research, the Bayesian nonparametric approach is an alternative to parametric modeling and selection. The Dirichlet Process (DP) is a stochastic process used in Bayesian nonparametric models of data, particularly in infinite mixture models, and is currently one of the most popular Bayesian nonparametric models [20]. Here, we use DP to find the correct numbers of mixture components in two Markov chains for the pairwise market coupling process. The results reveal that the numbers of mixture components in the two Markov chains are all equal to two.
- *Specification of the Logistic Regression Elements.* As mentioned in Section IV, there is a time window w for the forecasting process, according to the domain knowledge and several tests in the experiments, here we set w equal to two and three.

B. Comparative Methods

- LR-(E, C): This model forecasts crisis based on selected indicators of Equity market and Commodity



(a) DJIA (b) Gold (c) WTI Oil

(d) TED Spread (e) BAA Spread

Fig. 5: Indicators Returns Behavior Over Time

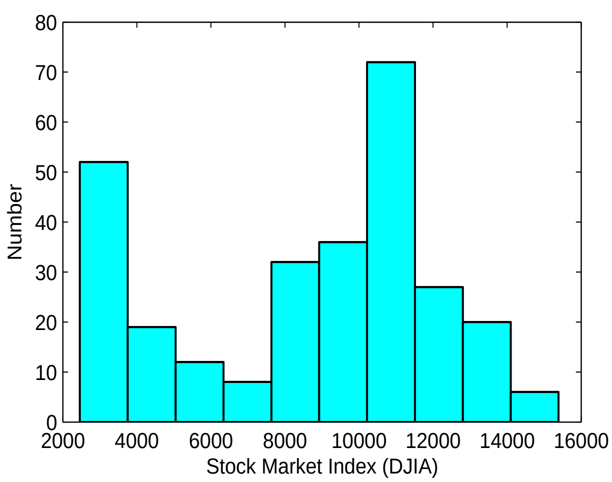


Fig. 6: The Distribution of DJIA

market directly, without considering the hidden complex market couplings.

- LR-(C, I): This model forecasts crisis based on selected indicators of Commodity market and Interest market directly, without considering the hidden complex market couplings.
- LR-(E, I): This model forecasts crisis based on selected indicators of Equity market and Interest market directly, without considering the hidden complex market couplings.
- LR-$\mathbb{C}(E, C)$: This model forecasts crisis based on market couplings from Equity market and Commodity market.
- LR-$\mathbb{C}(C, I)$: This model forecasts crisis based on market couplings from Commodity market and Interest market.
- LR-$\mathbb{C}(E, I)$: This model forecasts crisis based on market couplings from Equity market and Interest market.

C. Performance Metrics

As discussed in Section IV, $\Gamma^{(i,j)}$ is a fit function to evaluate the performance of market couplings in forecasting crisis, below we introduce several evaluation metrics as the fit function.

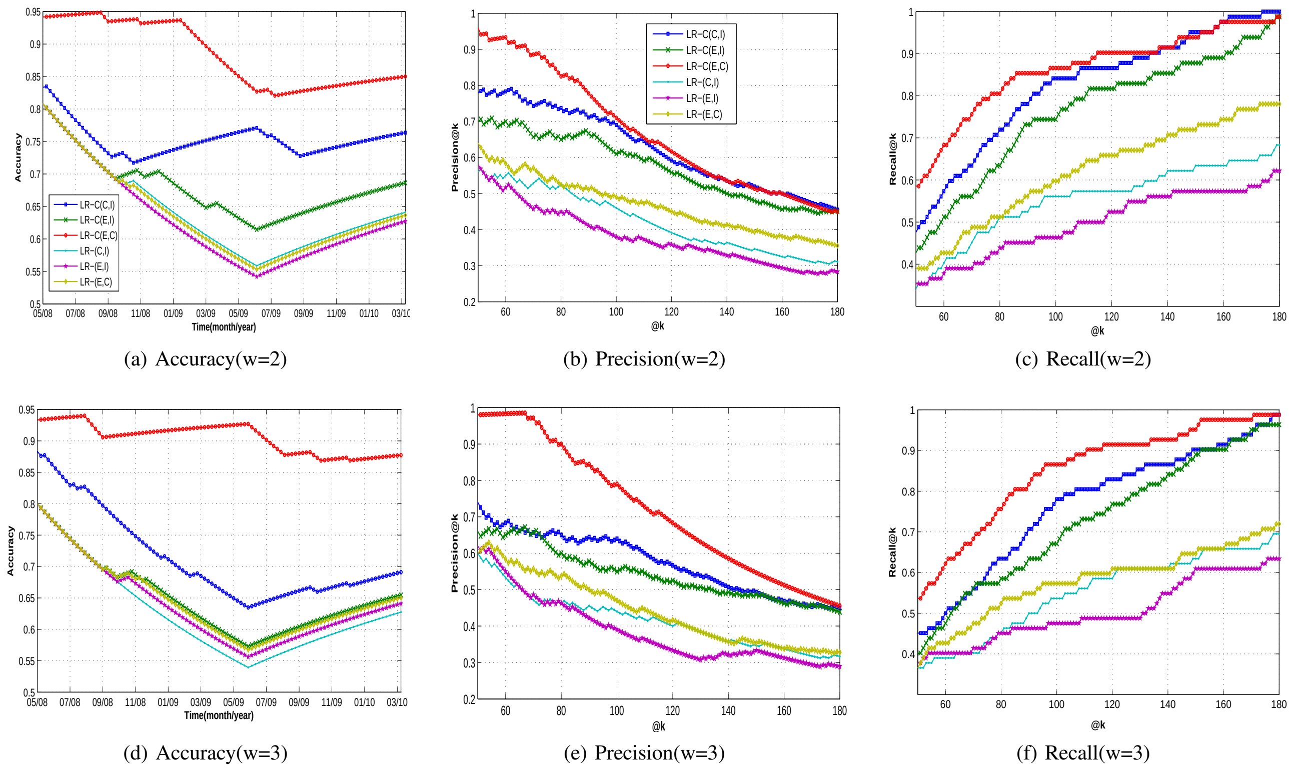
- *Accuracy*. Accuracy is the percentage of correctly classified instances.

$$Accuracy = \frac{TN + TP}{TP + FP + FN + TN} \tag{20}$$

where TP, TN, FP and FN represent true positive, true negative, false positive and false negative, respectively. We treat the financial crisis cases as the positive class here

- *Precision*.

$$Precision = \frac{TP}{TP + FP} \tag{21}$$



(a) Accuracy(w=2) (b) Precision(w=2) (c) Recall(w=2)

(d) Accuracy(w=3) (e) Precision(w=3) (f) Recall(w=3)

Fig. 7: Technical Performance of Various Approaches

- *Recall.*

$$Recall = \frac{TP}{TP + FN} \tag{22}$$

D. Forecasting Results

Here, we evaluate the financial crisis forecasting abilities of three pairwise market couplings based methods and three pairwise market indicators based methods on the testing period with window size $w = 2$ and $w = 3$. Accuracy, precision and recall in the former part are calculated. The results are illustrated in TABLE III and Fig. 7.

TABLE III: Accuracy Performance of Various Approaches

Approach	Accuracy	
	$w = 2$	$w = 3$
LR-(E, C)	0.6800	0.6727
LR-(C, I)	0.6840	0.6960
LR-(E, I)	0.6720	0.6827
LR-$\mathbb{C}(E, C)$	**0.8640**	**0.8584**
LR-$\mathbb{C}(C, I)$	0.7920	0.7227
LR-$\mathbb{C}(E, I)$	0.7240	0.6920

TABLE III shows the accuracy performance of the six approaches over the whole testing period. From the table we can see that the market couplings based approaches perform better than the approaches without considering market couplings. For instance, the LR-$\mathbb{C}(C, I)$ has about 10% improvement over the LR-(C, I) when time window equals to 2, and the LR-$\mathbb{C}(E, C)$ has around 16% gain over the LR-(E, C) when time window equals to 3. These illustrate that the pairwise market couplings have higher relations with financial crisis when compared with those simple indicators. The reasons can be interpreted as following: 1) the pairwise couplings is the "essence" of market contagion, which means that the pairwise couplings can better reflect the financial crisis; 2) in this paper we consider two different types of couplings (intra-market couplings and inter-market couplings) which can represent the pairwise market couplings well; 3) the CHMM is demonstrated as a useful tool to capture the complex hidden couples between pairwise markets.

Fig. 7 shows the technical performance of accuracy, precision and recall by setting two different window sizes, where the horizontal axis (@k) in Fig. 7b 7c 7e 7f stands for the number of predicted trading weeks in financial crisis, and the vertical axis represents the values of technical measures. We can easily find that our market couplings based approaches have better performance under all evaluation metrics. For example, recall represents the probability that a crisis is retrieved, the recall improvement of LR-$\mathbb{C}(C, I)$ could be as high as 20% against LR-(C, I) at k=140 while the window size is 2. In addition, as shown in Fig. 7b 7e, the three market couplings based approaches achieve higher precision with any k.

Interestingly, we can find from the table and figures that results from pairwise market couplings are conflicting with each other. Specifically speaking, the market couplings from Equity market and Commodity market ($\mathbb{C}(E, C)$) performs better than other two market couplings $\mathbb{C}(E, I)$ and $\mathbb{C}(C, I)$ under accuracy metric, while the performance under precision

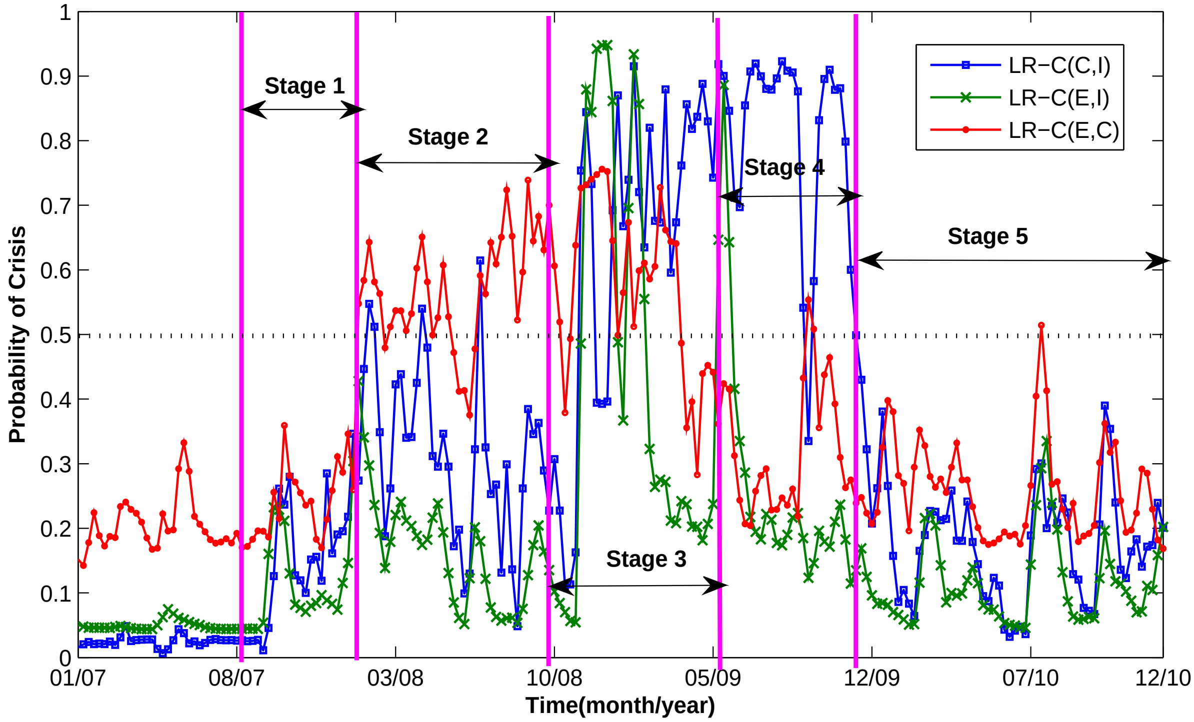


Fig. 8: Market Couplings Behavior during 2008 Financial Crisis ($w = 2$)

and recall do not reveal too much difference, especially the results from LR-$\mathbb{C}(E, I)$ and LR-$\mathbb{C}(C, I)$. This may be because the pairwise couplings changed during different stages of financial crisis. Below we discuss the implications from pairwise market couplings on crisis periods identification.

E. Crisis Periods Identification

In this part we discuss the three pairwise market couplings behavior during the 2008 financial crisis periods: couplings between Equity market and Commodity market ($\mathbb{C}(E, C)$), couplings between Commodity market and Interest market ($\mathbb{C}(C, I)$) and couplings between Equity market and Interest market ($\mathbb{C}(E, I)$). As depicted in Fig. 8 (the vertical axis represents probabilities of crisis predicted by pairwise market couplings, and the horizontal axis represents the time), according to the three pairwise market couplings, the whole period can be divided into five stages: Stage 1 is a stage of "crisis launch" and spans from August 2007 to December 2007, in this period the probabilities of crisis forecasted by all three pairwise couplings begin to grow. Stage 2 is defined as "$\mathbb{C}(E, C)$" stage, where the couplings from Equity market and Commodity market has a sharp increase in this stage (December 2007 to September 2008). A possible explanation is that crisis always first revealed by Equity market and Commodity market, the Equity market is always considered as risky market while the Commodity market is the opposite. Stage 3 is described as "sharp fluctuation" stage, where the all pairwise market couplings reveal high financial crisis probabilities (September 2008 to April 2009). This maybe caused by the spread news of crisis and shifts in investors' common but changing appetite of risk. Stage 4 is a "$\mathbb{C}(C, I)$" stage spans from April 2009 to November 2009. An explanation is at this stage the macro-control measures (e.g. cutting rate) begin to take effect. Stage 5 is described as "post-crisis" while the behaviors from all the pairwise couplings become stable (after November 2009).

Interestingly, the crisis periods identification obtained above is consistent with former research [21] and official lines provided by the Bank of International Settlements [22]. Specifically speaking, the identified Stage 1 and Stage 2 are corresponding to the "initial financial turmoil" phase illustrated in [22]; Stage 3 match with "deterioration" phase; Stage 4 is corresponding to "stabilization and tentative signs of recovery" phase. All these illustrate the great importance of market couplings in understanding financial crisis at different stages.

VI. Conclusion

This paper investigate the relations of financial crisis and various pairwise market couplings from three typical global financial markets: Equity market, Commodity market and Interest market. Specifically, a CHMM-LR framework is built to study the relations, where CHMM is imported to capture the complex hidden pairwise market couplings, and LR is applied to evaluate the crisis forecasting capabilities based on the couplings. The experimental results show that the proposed framework achieves satisfactory performance, and the pairwise market couplings is of great importance in understanding financial crisis.

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